Geometric and Quantum Methods for Information Retrieval

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Abstract

This paper reviews the recent developments in applying geometric and quantum mechanics methods for information retrieval and natural language processing. It discusses the interesting analogies between components of information retrieval and quantum mechanics. It then describes some quantum mechanics phenomena found in the conventional data analysis and in the psychological experiments for word association. It also presents the applications of the concepts and methods in quantum mechanics such as quantum logic and tensor product to document retrieval and meaning of composite words, respectively. The purpose of the paper is to give the state of the art on and to draw attention of the IR community to the geometric and quantum methods and their potential applications in IR and NLP.

1 Introduction

Search engine algorithms like PageRank and HITS exploit the link structure of the web, besides the lexical content in web pages (see e.g. [1]). Essentially they mine human decisions about what pages are good to link to for a particular subject. For many information spaces, however, this type of data is not available. Vector space retrieval has proven an effective substitute where the requisite behavioural data to support ranking algorithms is not present and is in widespread use in systems like Apache Lucene and (at least in early versions) Inktomi-derived systems. Vector space model retrieval methods are potentially richer and more sensitive than the key words matching.

In recent works vector space models have been extended to incorporate an apparently productive analogy with parts of the mathematical apparatus of quantum theory and the representation and manipulation of word meaning. Keith Van Rijsbergen in [10] established the analogies between information retrieval and quantum mechanics by extending the Euclidean space based vector model used in information retrieval (IR) to Hilbert space based one. Hilbert space is the mathematical foundation of quantum mechanics. It also discussed the applications of the standard theorems in quantum mechanics to address problems in IR such as relevance feedback and ostensive retrieval. Dominic Widdows in [12] explored the geometry for natural language understanding, in particular the word meaning and context. Some promising results have been obtained. For example, [7] used the tensor product to study the correlation of the user behaviour when accessing one document with the usefulness (or relevancy) of document for the user's task, namely the implicit feedback for IR. [11]

applied the quantum logic, in particular vector negation to document retrieval and lexical priming. For the latest developments of using quantum methods for information retrieval and natural language processing, see the presentations and papers of the International Quantum Interaction (QI) Symposium[1].

Actually vector space models of semantics are not new; see for example latent semantic indexing [3]. LSI suffered from high computational cost and the lack of incremental update capability which limited its applicability, but recent work on Random Indexing promises to solve these problems [6].

In this paper we review the recent developments in applying the geometric and quantum methods for IR and natural language processing (NLP). The main purpose is to draw attention of the IR community to the geometric and quantum methods and their potential applications in IR and NLP. We first explain the information retrieval and quantum mechanics and present the analogies between the two. We then discuss some quantum mechanics phenomena in information retrieval, and some recent applications of quantum mechanics methods to IR and NLP. In the last section we give some discussions and future works.

2 Information retrieval

An **Information retrieval (IR)** system finds relevant information from a collection of information objects. IR involves three elements:

- A collection of information objects, which may be documents, web pages, images, videos, audios, or RDF triples. The conventional IR system uses documents. Internet search engines search web pages, mainly based on the text content and/or hyper-links that web pages contain. Searching over multi-media objects such as images, video and audio is a new domain. Selecting the relevant RDF triples is a new task for semantic web applications [4]. Note that the collection may be closed, in the sense that the objects in it do not change, or dynamic, in the sense that new objects may be added.
- An information need from a user. It may be a simple query containing one or more terms, or a more complicated information need described by a paragraph of text, or some typical examples (in this case the user wants to obtain more objects from the collection which are similar to the given examples).
- The search procedure, which finds objects from the collection which are relevant to the information need.

Vector space models (VSMs) have been widely used in IR [1]. A VSM represents the objects for retrieval and the information query in the same multi-dimensional Euclidean space (or vector space). The coordinates of the vector space correspond to the content words (or terms) for text retrieval – one coordinate for each particular content word. The set of content words for one application can be pre-defined [11], or can consist of all the words contained in the documents except the function words and connectives (such as articles and prepositions, which are typically regarded as 'stopwords' in IR). Hence the vector space may be of very high dimensionality if there are many content words. For one document or query, the component of its vector representation for one particular coordinate is the number of occurrences of the term corresponding to that coordinate (*TF*), which may be modified by

[1]QI-2007 (http://ir.dcs.gla.ac.uk/qi2007/), QI-2008 (http://ir.dcs.gla.ac.uk/qi2008/), and QI-2009 (http://www-ags.dfki.uni-sb.de/ klusch/qi2009/index.html)

the inverse document frequency (*IDF*) of the term in some way, and the vector may be normalised.

The VSM measures the similarity between object and query by computing the cosine of the angle or inner product between the object vector and query vector. Then it ranks the objects according to the similarities to the query and presents the ranked list to the user.

Note that only part of user's information need is expressed through the query. A user may give the IR system feedback about which objects are relevant and irrelevant, and the system may use the feedback to refine the query and make a new search. The VSM can handle the relevancy feedback and query refinement by using Rocchio's method [1], for example. User feedback is one form of the interaction between users and IR systems. In another form, the user changes the information need after obtaining more information by e.g. reading some of the retrieved documents.

The changes in both the collection and the information need reflect the dynamic nature of IR. An IR system should address those issues.

The VSM uses the Euclidean space, a concept from geometry. Based on the VSM, Keith van Rijsbergen proposed the **geometry of IR** in [10], which uses the Hilbert space. In fact only the inner product of two vectors is important in the VSM. The mathematical space with inner products is Hilbert space. Therefore the VSM method can be generalised to a Hilbert space.

Hilbert space is more abstract (or general) than Euclidean space. For example, Hilbert space may have a finite or infinite dimension and can use real or complex numbers, while a Euclidean space has only finite dimension and uses real numbers. Hence the Hilbert space may provide more possibilities for improving IR methods. For example, [10] mentioned to use complex numbers instead of real numbers in the vector representation, and also proposed to regard a retrieval object as an abstract object (or element) in Hilbert space and a set of bases in the space as corresponding to one particular point of view to represent the retrieval objects.

Moreover, as Hilbert space is the mathematical foundation for quantum mechanics (QM), basing IR on Hilbert space creates an analogy between IR and QM and may usefully bring some concepts and methods from QM into IR.

3 Quantum Mechanics

Quantum mechanics (QM) is the study of mechanical systems whose scales are close to or below the atomic scale, such as molecules, atoms, electrons, protons and other subatomic particles. QM reveals some important properties of quantum system that are very different from the normal macroscopic mechanical system that the classical Newtonian mechanics studies and we experience with in our daily life. We will discuss some of the properties later on.

QM is based on complex Hilbert space. A quantum system is associated with a complex separable Hilbert space. The possible states of the system are represented by unit vectors in the Hilbert space. An observable (or measurable properties) of the quantum system, such as energy, position, and momentum, corresponds to a self-adjoint linear operator in the Hilbert space[2]. The possible values of the observable are the eigenvalues of the corresponding self-

[2] It is worth noting that QM asserts only the existence of the correspondence between a quantum system and a Hilbert space, but does not provide a general solution for finding out the corresponding Hilbert space for any

adjoint linear operator[3]. Note that a linear operator may have discrete (and even limited) eigenvalues, meaning that the corresponding observable has only discrete values. During a measurement for one observable, the system collapses from a given initial state to a particular eigenstate corresponding to one eigenvector of the linear operator for the observable. The probability of collapsing to the eigenstate is the square of the absolute value of the inner product between the initial state and the eigenstate. The value of the observable for the measurement is the eigenvalue associated with the eigenvector (eigenstate).

The Hilbert space of a composite system is the Hilbert space *tensor product* of the state spaces associated with the component systems. The state of a composite system may or may not be the tensor product of the states of the component systems. If it is not, it is an entangled state, as explained below.

QM incorporates the following four types of phenomena that classical physics cannot account for.

- the quantization (discretization) of certain physical quantities, in the case that the associated self-adjoint linear operator has discrete eigenvalues.
- wave-particle duality, a concept that all forms of matter and energy exhibit wave-like properties (such as filling in the space and having a certain wavelength) and particle-like ones (such as having a position in space).
- the uncertainty principle, which states that locating a particle in a small region of space makes the velocity of the particle uncertain; and conversely, that measuring the velocity of a particle precisely makes the position uncertain. This principle can be explained by the wave-particle duality. In fact the uncertainty principle holds for any two observables if the associated self-adjoint operators A and B are non-commutative, namely $AB \neq BA$.
- quantum entanglement, a phenomenon in which the quantum states of two or more objects are linked together — even though the individual objects are spatially separated. This interconnection leads to correlations between observable physical properties of remote systems.

Some of the QM phenomena have been found in IR and other information processing tasks. Before we describe those phenomena, we will discuss the analogy between QM and IR.

4 Analogy between QM and IR

There are some interesting correspondences between the components of QM and IR, as summarised in Table 1. QM uses a complex Hilbert space to represent a particular quantum system, while IR needs some information space to represent the objects in one collection for retrieval. The state vector is the complete and maximal summary of the characteristics of the quantum system at a moment in time, which changes at different time. Objects in collection contain all the information available for retrieval. We may make an analogy of one object in

quantum system. Given a quantum system and some observable, you have to work out the particular Hilbert space corresponding to the quantum system and the particular self-adjoint linear operator for the observable.

[3] It has been proved mathematically that all the eigenvalues of a self-adjoint linear operator in complex Hilbert space are real numbers, which guarantees that the values of an observable are all real numbers.

Table 1: Analogy between QM and IR.

QM	IR
a quantum system	a collection of object for retrieval
complex Hilbert space	information space
state vector	objects in collection
observable	query
measurement	search
eigenvalues	relevant or not for one object
probability of getting one eigenvalue	relevance degree of object to query

Table 2: Comparison of VSM with QM.

QM	VSM for IR
complex Hilbert space	real Euclidean space
state vector	vector
self-adjoint linear operator for observable	query vector
measurement – interaction between measurement device and quantum system	search – may involve interaction between user and system.
eigenvalues of the operator	relevant or not for one object
probability of obtain one eigenvalue	relevance degree of object to query

collection as a state vector of the quantum system at one particular time (or the quantum system at one particular state). Observables are the physical quantities that one can measure on quantum system. Different quantum systems may have different types of observables. Query represents a question for which a user may obtain answer from one collection. Different collections have different types of sensible questions to ask. Measurement is a procedure to determine the value of an observable for one quantum system in one particular state, while search is a procedure to determine the relevance of an object in collection to a query.

Since the VSM is a common method for IR, Table 2 compares the VSM with QM, based on the analogy between IR and QM discussed above. We can see that the QM use more general model than the VSM, which may inspire new methods beyond the VSM. For example, complex Hilbert space is more general and expressive than the Euclidean space. We may use complex numbers to represent quantities in IR. Hilbert space will enable us to represent the retrieval objects as abstract objects and to use any base and base transformation. In particular one may construct some base according to the query and use the base to represent each object from query's point of view. One may also represent the query using some subspace or operator in Hilbert space.

5 QM Phenomena in IR

Since the QM is based on Hilbert space, and the Euclidean space used in the VSM of IR and natural language processing (NLP) is a specific type of Hilbert space, it is not surprising that there are QM phenomena in IR and NLP.

[9] used a recommender system as example to demonstrate some QM phenomena in data analysis. As we know, such a system recommends the similar new objects (e.g. films, books) to the user based on his previous experience.

- In the VSM, the similarity of a new object to the previously seen objects is computed based on the correlation matrix of the vectors of the previously seen examples. The correlation matrix is a self-adjoint linear operator in the vector space, corresponding to an observable in QM.
- [9] obtained a Bell's inequality of similarities and found some examples violating the inequality. Violation of Bell's inequality means that one cannot predict the future using the user's profile derived from the past in the framework adopted. It can be viewed as an indicator of a quantum statistical correlation.
- [9] also pointed out that the concept lattice based on the taxonomy of concepts is not distributive, namely,
$$x \wedge (y \vee z) \neq (x \wedge y) \vee (x \wedge z)$$
For example, in a taxonomy of animals, take $x =$"bird", $y =$"human" and $z =$"lizard". Then both $x \wedge y$ and $x \wedge z$ are empty, so that $(x \wedge y) \vee (x \wedge z)$ remains empty. On the other hand, $y \vee z =$"vertebrates", because vertebrates are the smallest class including both humans and lizards. Hence $x \wedge (y \vee z) =$ "bird", which is not empty. The non-distributive property does not hold in the classic set based logic but occurs in quantum logic (which will be discussed in the following section).

[8] discussed several psychological experiments for human's word association and described some interesting findings from the experiments. In one experiment, giving a cue word to a human, they measured the likelihood that she recall one related word called target word. Note that one word may have several associative words. For example, given the word "planet", a human may recall the related words such as "earth", "moon" and "universe". Figure 1 shows hypothetically a cue word, a target word and two associative words with the pre-existing links. There are three links from cue word to the target word and two associates, one associate-to-associate link, and one associate-to-target link from the Associate 2 to target. The values on the links indicates relative link strengths. Besides the links between the cue word and target word, the associate-to-associate links are also useful for the recall of target word from cue word. One explanation for why associate-to-associate links benefit recall uses the spreading activation equations

$$R(T) = S_{ct} + \sum_{i=1}^{n} S_{ci} S_{it} + \sum_{i=1}^{n} \sum_{j=1}^{n} S_{ci} S_{ij} S_{jt} = 0.8 + 0.1 * 0.7 + 0.2 * 0.6 * 0.7 = 0.954$$

which is based on the classic idea that activation spreads through a fixed associative network, weakening with conceptual distance by multiplying link strengths. Another explanation is based on the assumption that each link in the associative network contribute additively to the recall strength. It uses the activation at a distance equation to compute the recall likelihood

$$R(T) = S_{ct} + \sum_{i=1}^{n} S_{ci} + \sum_{i=1}^{n} S_{it} + \sum_{i=1}^{n} \sum_{j=1}^{n} S_{ij} = 0.8 + 0.2 + 0.1 + 0.7 + 0.6 = 2.4$$

It was shown in [8] (in which the target word was the same as the cue word and $S_{ct} = 0.0$) that the psychological experimental results were much more consistent with the second

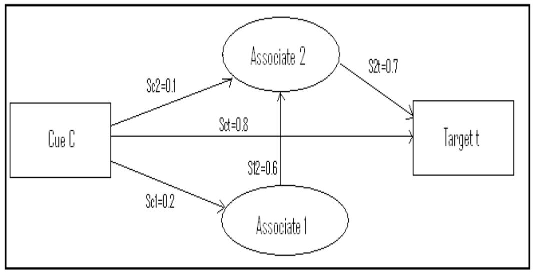

Figure 1: A hypothetical cue and target with two associates, and cue-to-target link, cue-to-associate links, associate-to-associate link, and associate-to-target links with link strengths (modified from [8]).

explanation than the first one. The authors explained the additive effects of associate-to-associate and associate-to-target links on the recall level by the entanglement of the target and its associates. [8] speculated that there were entanglements between the target and its associates and because of the entanglements, the target word and the associated words acted as correlated instead of separated entities.

6 Quantum Logic and its application in document retrieval

Quantum logic is defined on the sub-spaces of Hilbert space (see [15], for example). It is also called vector logic. Given a Hilbert space $\mathcal{H}$, the logical connectives of the quantum logic defined in the sub-spaces of $\mathcal{H}$ are

- The negation of a sub-space A is the orthogonal sub-space of A.
- The conjunction of the two sub-spaces A and B is the sub-space consisting of the vectors belonging to both A and B.
- The disjunction of the two sub-spaces A and B is the sub-space consisting of the vectors each of which is a linear combination of two vectors respectively from A and B.

In comparison with classic set logic (namely considering one sub-space as a sub-set of the vector set $\mathcal{H}$), the conjunction is the same but the negation and disjunction are different in the two logics. In set logic the negation of one sub-set contains all the elements not being in the sub-set, while in quantum logic the negation of one sub-space contains only the vectors that are orthogonal to the sub-space. The conjunction of the two sub-sets in set logic is the union of the elements in each of the two sub-sets, while the conjunction of the two sub-spaces in quantum logic contains not only all the vectors of the two sub-spaces but also those vectors between the two sub-spaces. In other word, the quantum logic utilizes the structure of Hilbert space, whereas the set logic just uses the set relationships among the sub-sets of Hilbert space.

Quantum logic can be used in the VSM based methods. VSM has been widely used in IR and NLP applications. [11] applied the orthogonal negation in vector space for modelling word-meaning and document retrieval and compared the results with those using set logic. In the following we discuss the document retrial experiments presented in [11].

For document retrieval the experiments were done on the British National Corpus, one news article corpus, and one medical document corpus. To obtain vector representation for document and query, 1000 content words were selected. Each document was represented as a 1000-dimensional vector — each coordinate corresponded to one content word and the tf-idf weighting scheme was used for the value of each coordinate. A query had the form $a\ NOT\ b$, where a and b are two terms. Each term was also represented as a 1000-dimensional vector by counting the occurrences of the selected content words in a 15 words context window centred around the term in document and using the tf-idf weighting scheme. Each vector was normalised. Suppose that the terms a and b have the vectors v_a and v_b respectively, according to the orthogonal negation, the query $a\ NOT\ b$ has the vector representation

$$v_q = v_a - < v_a, v_b > v_b$$

Then the document retrieval was performed by computing the similarity between the query vector and document vectors and ranking documents according to the similarity.

The author also implemented the document retrieval using the negation of set logic. It first retrieved the documents by using the term vector v_a and then removing those documents containing the unwanted term b.

Experimental results showed that, in comparison to the set logic negation, the vector negation method retrieved the documents containing a much lower number of negative synonyms and negative neighbours but, unfortunately, also a lower number of positive terms. For more detailed discussions about the experimental results, see [11].

7 Tensor product and its application in combination of meaning of individual words

QM represents composite quantum system using the tensor product of the Hilbert spaces of individual quantum systems. The tensor product of two objects consists of the combinations of any two components of the two objects, respectively. For example, given two matrices **A** and **B**

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

the tensor product of **A** and **B** is

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_{11}\mathbf{B} & a_{12}\mathbf{B} \\ a_{21}\mathbf{B} & a_{22}\mathbf{B} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} & a_{11}b_{12} & a_{12}b_{11} & a_{12}b_{12} \\ a_{11}b_{21} & a_{11}b_{22} & a_{12}b_{21} & a_{12}b_{22} \\ a_{21}b_{11} & a_{21}b_{12} & a_{22}b_{11} & a_{22}b_{12} \\ a_{21}b_{21} & a_{21}b_{22} & a_{22}b_{21} & a_{22}b_{22} \end{bmatrix}$$

Similarly, the tensor product of two vectors is a matrix (or a long vector).

In the VSM methods for IR and NLP application, we often need the composite of two or more elements. [2] investigated the composite meaning of a sentence using the semantic representations of individual words. The method used was the tensor product of the semantic vectors of individual words. For example, given a sentence "John likes Mary", and the vector representation of the semantics of the three words, v_{John}, v_{likes}, and v_{Mary}, the sentence was represented as tensor product

$$v_{John} \otimes_{sv} v_{likes} \otimes_{vo} v_{Mary}$$

Table 3: Verb noun similarities for several word pairs using three different vector composition methods.

First Pair	Second Pair	Vector Sum	Direct Sum	Tensor Product
earn money	pay wages	0.66	0.41	0.16
earn money	wages pay	0.66	0.51	0.26
eat apple	eat tomato	0.73	0.65	0.29
eat apple	throw apple	0.57	0.52	0.04

where $\otimes_{sv}$ and $\otimes_{vo}$ represent tensor products for subject and verb and for verb and object, respectively. Using the tensor product operations on semantic vectors, one sentence is represented as a matrix. Then we may compute the similarity of two sentences by using the inner product of the two matrices.

[13] used the vector composition to model semantic composition. In one experiment the author computed the similarity between verb-noun pairs using vector composition methods. It is expected that *eating a tomato* is relatively similar to *eating an apple*, whereas *throwing an apple* is quite different. This experiment first represents each word as a semantic vector using the 1000 content-bearing words, as described in Section 6. Then it computed a vector for a pair of words using three different methods. Given the two vector a and b with the same dimension n, the vector addition operation on them resulted in a vector of the dimension n, $a+b$. The direct sum of the two vectors was the concatenation of the two vectors, a vector of the dimension $2n$. The tensor product of the two vectors was a vector of dimension $n*n$. As one pair of words was represented by a vector, the similarity of two pairs can be computed as the inner product of two corresponding vectors.

Table 3 shows some results of the comparison of the similarities of noun verb pairs. We can see that the tensor product numbers gave the best discrimination.

8 Conclusion and future work

The geometry in Hilbert space provides a general framework for IR. Using the framework, IR can be studied from a broader view. There are many similarities between IR and QM. They both are based on Hilbert space and involve the interaction between system and user (or observer). From physics' point of view, QM studies the phenomena at sub-atomic level, which are very different from those physical phenomena at a human perception level. However, QM phenomena, such as entanglement, the uncertainty principle and state collapse, exist in information-rich areas such as information processing, biology, geosciences and economics [14]. The concepts and methods in QM may inspire new concepts and methods in those areas. For example, as discussed earlier, quantum logic and the tensor product operation have been used in IR and NLP and obtained promising results. [5] proposed a theoretical framework for representing documents, inspired by physical measurement on quantum state, and their potential uses in clustering documents.

On the other hand, the quantum system is simpler than IR and natural language understanding because it involves a small number of types of elements and a few operations. In order to model the complicated phenomena in IR and natural language, one may have to employ more complicated mathematical tools. Differential geometry studies the objects with a more complicated structure than Hilbert space. In particular, it does not presuppose the existence of a global set of coordinates or attributes, is not constrained to being flat or linear, and pays much attention to the local context. Hence it may be more suitable for modelling the meaning of text [12]. For example, instead of specifying some attributes as global coordinates to represent all queries and retrieval objects, the information space for IR (or semantic space for NLP) may be segmented into different areas in a hierarchical fashion to represent different contexts or semantic types in natural language, and each context area is characterised by the attributes and structure relevant to the context.

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